The Evolution of Interpersonal Relationships in a Social Group

Bo Hu, Xin-Yu Jiang, Bing-Hong Wang, Jun-Feng Ding, Tao Zhou, and Yan-Bo Xie*
Nonlinear Science Center and Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, PR China

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A model has been proposed to simulate the evolution of interpersonal relationships in a social group. The small social community is simply assumed as an undirected and weighted graph, where individuals are denoted by vertices, and the extent of favor or disfavor between them are represented by the corresponding edge weight. One could further define the strength of vertices to describe the individual popularity. The strength evolution exhibits a nonlinear behavior. Meanwhile, various acute perturbations to the system have been investigated. In the framework of our model, it is also interesting to study the adaptive process of a new student joining a class midway.

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I. INTRODUCTION

A social network is a set of people with some pattern of contacts or interactions between them[1, 2]. Cases that have been studied include the patterns of friendships between individuals[3, 4], business relationships between corporations[5, 6], and intermarriages between families[7]. In a social group, all individuals will have their friends and enemies. Rapoport and others have once studied friendship networks of school children[4**?**], due to its relatively simple pattern and small sample size. Recently, Alain Barrat, et al. have proposed a general model for the growth of weighted networks[**?**], considering the effect of the coupling between topology and weights' dynamics. It appears that there is a need for a modelling approach to complex networks that goes beyond the purely topological point. We have proposed a simple model to simulate the interpersonal relationships among pupils of a class[8]. In accordance with empirical observations, similar human relations between people will promote friendship, while opposite interpersonal relations may lead to hostility. The first impression effect, as shown in [8], has played a significant role in the evolution of the interpersonal relations. As a further research and development of our original work, this paper, with both analytical and experimental methods, gives a detailed analysis to the evolution behaviors. The reactions of the system to various perturbations are also investigated.

II. REVIEW THE MODEL

The model system comprises N students and we introduce a $N \times N$ weighted adjacency matrix to describe its interpersonal relationships. The symmetric matrix elements ω_{ij} represent the weight of edge e_{ij}, where i, j=1, 2, …, N. The value of ω_{ij} is discrete and can be negative for the cases of disfavor relationships; each contact between individuals are supposed to alter weight by ± 1 at most. This assumption makes the pairwise interaction moderate; thus, love or hatred in this model is not formed in seldom contacts. In the original model, we assign value 1 with probability p, and -1 with probability 1-p to the elements of matrix for initialization, where p is called *the initial amity possibility.* The definition of the model is based on the weights' dynamics:

(i)First, suppose student i has been randomly selected from the class. Then, he takes the initiative in contacting student j with possibility:

$$W_{i\to j} = \frac{|\omega_{ij}|+1}{\sum_{j=1,j\neq i}^{N}(|\omega_{ij}|+1)}. \tag{1}$$

Obviously,

$$W_{i\to j} = W_{j\to i} \tag{2}$$

The reason to choose this possibility is to avoid the scenario that student j with ω_{ij}=0 cannot be selected by student i, (for more information, please see Ref[8]).

(ii)Now, i and j have been chosen for interaction. Since "birds of a feather flock together" and similar human relations and social environments are likely to promote friendship, we could define γ_{ij} as below to describe the interpersonal relation similarity:

$$\gamma_{ij} = C^{-1}\sum_{\alpha}\omega_{i\alpha}\cdot\omega_{\alpha j} \tag{3}$$

where

$$C = \sqrt{\sum_{\alpha}\omega_{i\alpha}^2}\cdot\sqrt{\sum_{\beta}\omega_{j\beta}^2} \tag{4}$$

Apparently, γ_{ij}=γ_{ji} and $-1 \le \gamma_{ij} \le 1$.

*Electronic address: bhwang@ustc.edu.cn, Fax:+86-551-3603574.

Then, the rules of revolution are (for more details, please see our original work[8]):

when $\gamma_{ij} \geq 0$

$$\omega_{ij} \to \omega_{ij} + 1, \omega_{ji} \to \omega_{ji} + 1 \tag{5}$$

with possibility γ_{ij}, and nothing is altered with possibility $1 - \gamma_{ij}$;

when $\gamma_{ij} < 0$

$$\omega_{ij} \to \omega_{ij} - 1, \omega_{ji} \to \omega_{ji} - 1 \tag{6}$$

with possibility $|\gamma_{ij}|$, and nothing is altered with possibility $1-|\gamma_{ij}|$.

After the weights have been updated (*the symmetry requirement of the matrix must be satisfied in every update), the process is iterated by randomly selecting a new individual for the next contact until the class disbands.

The most commonly used topological information about vertices is their degree and is defined as the number of neighbors. A natural generalization in the case of weighted networks is the strength. Define strength s_i of node i as below to describe individual popularity:

$$s_i = \sum_{j=1}^{N} \omega_{ij}. \tag{7}$$

We could write the evolution equation of strength s_i as bellow:

$$\frac{\partial s_i}{\partial t} = \sum_{j=1, j \neq i}^{N} \frac{1}{N} W_{i \to j} \gamma_{ij} + \sum_{j=1, j \neq i}^{N} \frac{1}{N} W_{j \to i} \gamma_{ji} \tag{8}$$

The first term on the right hand side of Eq. (8) describes all the possible alterations to s_i at time step t, contributed by the cases that i is first randomly chosen and then takes initiative in contacting j; likewise, the second term describes all contributions from j choosing i. Given that $W_{i \to j}$=$W_{j \to i}$ and γ_{ij}=γ_{ji}, we could simplify master Eq. (8):

$$\frac{\partial s_i}{\partial t} = \frac{2}{N} \sum_{j=1, j \neq i}^{N} W_{i \to j} \gamma_{ij} \tag{9}$$

$$= \frac{2}{N} \frac{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1) \gamma_{ij}}{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1)}. \tag{10}$$

Weight ω_{ij} has a certain distribution in the final state (as shown in Ref.[8]), so do $W_{i \to j}$ and γ_{ij}. For convenience, we define:

$$< \gamma >_i = \frac{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1) \gamma_{ij}}{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1)}, \tag{11}$$

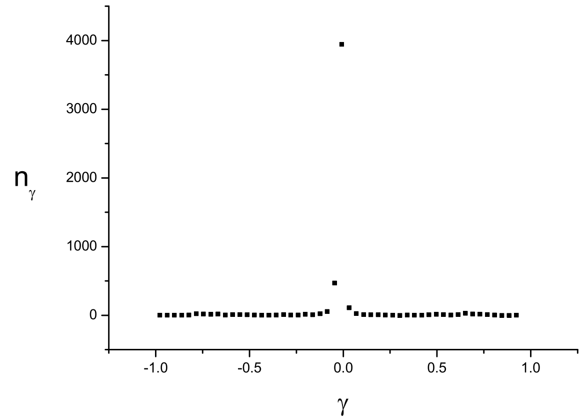

FIG. 1: γ distribution for p=0.00, N=100 and M=50 after 1.0×10^6 time steps.

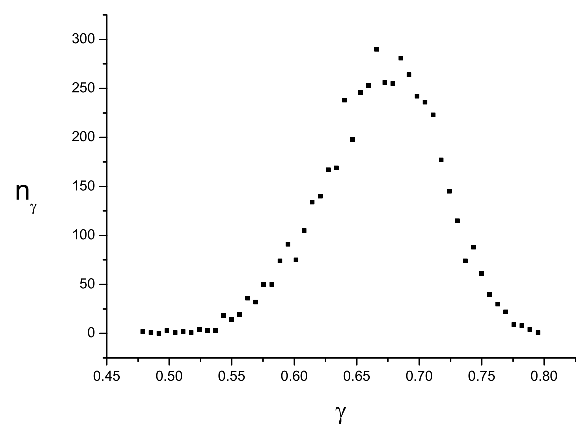

FIG. 2: γ distribution for p=1.00, N=100 and M=50 after 1.0×10^6 time steps.

obviously, $| < \gamma >_i | < 1$.

To obtain the distribution of γ, the range of γ is equally divided by M. Then, the range $[\gamma_{min}, \gamma_{max}]$ becomes $[\gamma_1, \gamma_2)$, $[\gamma_2, \gamma_3)$, ..., $[\gamma_M, \gamma_{M+1}]$, where γ_1=γ_{min}, γ_{M+1}=γ_{max}. Define n_{γ_l} as the number of γ between $[\gamma_l, \gamma_{l+1})$, l=1, 2, ..., M; when l=M the interval is $[\gamma_M, \gamma_{M+1}]$. As shown in Fig. 1, the distribution of γ has a pinnacle at γ=0. The average value of γ is -0.00118 as calculated. Since now the weight distribution also behaves a pinnacle near ω=0[8], it indicates that when the initial non-diagonal elements(weights) are all -1, the system behaves a tardy evolution. Many contacts will give no alteration to the weights, or equally, the effective contacts between people that change their relations occur infrequently. From Fig. 2, one can see that the γ distribution exhibits a convexity and reaches the maximum near γ=0.67. $\gamma > 0.47$ and the average of γ: $< \gamma >$=0.66817. Since the weight distribution $n_\omega \sim \omega$ also displays a con-

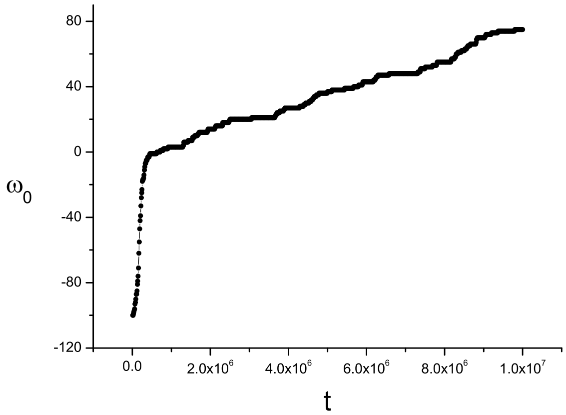

FIG. 3: weight evolution for N=100 after 1.0×10^7 time steps. Initially, all the weights are set +1, except ω_0=-100, where ω_0 denotes the designated negative weight.

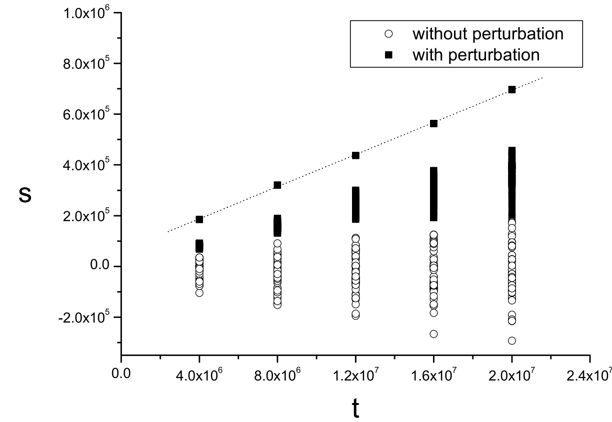

FIG. 4: strength evolution for N=50, M=50 and p=0.00 after 2.0×10^7 time steps. The dot line (with slop 0.03167) fits the strength evolution of the leader, and blacksquare(■) represents the strength of others, circle(○) denotes the strength evolution without the leader, under the same conditions. The data are drawn out at intervals of 4.0×10^6 time steps.

vexity at $\omega>0$, the effective contacts between individuals are quite frequent. Above results would be reviewed in the next section.

III. PERTURBATIONS AND EVOLUTIONS

In the original model, we suppose that individuals at first are not familiar with each other. The initial configuration of the system is determined by the initial amity possibility p. It can be interpreted as the first impression among them. However, it is completely possible that two individuals have good fellowship or a deep grievance previously. Plotted in Fig. 3 is $\omega_0\sim t$ for the process of an old grievance thawing in an initial harmonious group ($p\approx1.00$). ω_0, which denotes the old grievance between two from the very beginning, rises sharply from -100 to 0, and increases gently afterwards. This phenomenon can be interpreted from two aspects. On the one hand, the new environment provides them sufficient opportunities to contact other people, so they would not encounter frequently and aggravate their greivance. On the other hand, they both will get on well quickly with new individuals, due to $p\approx1.00$ (see Ref.[8]). Thus, after sufficient contacts they will have common friends who will play a conciliatory role in mitigating their old grievance.

All too often, a group has its leader who has great reverence among his people. He may be an elder chief, a captain of a football team or a chairman of an association. The other members of the group may have some conflicts and grievances. We are interested in the role of the leader playing in the development of the collective. For simplicity, we could assign a large integer to the matrix elements at row i (ω_{ij}=ω_{ji} and ω_{ii}=0) to denote the leader's relations, and set all the other non-diagonal elements as -1. In Fig. 4, we have taken this integer as 100. Apparently, the leader's strength grows linearly with the passage of time, while the strength of his members increases with a relatively slow velocity. By comparison with the evolution containing no leader, one could conclude that the strengths of the system have been boosted up by the leader. The slope of the leader's popularity evolution is 0.03167. In section 2, we have given that:

$$\frac{\partial s_i}{\partial t}=2<\gamma>_i/N. \tag{12}$$

For the case containing the leader: if we assume that $\omega_{kj}\ll\omega_{ij}$ for all j and $i\neq k$, then $\gamma_{ij}\approx1$, and further $\partial s_i/\partial t$=$2/N$=0.04, in approximate agreement with the experimental results. For the case absent the leader: the average speed of their popularity growth can be also estimated. If we suppose $<\gamma>_i\approx<\gamma>=-0.0118$ (as mentioned in section 2), then Eq. (12) gives the average velocity of popularity growth -7.2×10^4, which is quite low. But as one can see in Fig. 4, their strengths are dispersed at a certain time (nonlinear property). Note that the master equation (8) can only give an average rate of popularity growth. The leader has changed the possible state of disunity and make his men draw together.

It is also interesting to study the opposite case. We could put a public anti into a united group to see the strength evolution of the system, see Fig. 5. For initialization, we assign -100 to the matrix elements at row i (ω_{ij}=ω_{ji} and ω_{ii}=0) to represent the anti's relations, and set all the other non-diagonal elements as 1 to describe the initial public. Obviously, the anti's popularity decreases linearly, while the strength of the public rises at a relatively slow speed (approximate linear). By comparing with the case with no anti, one could see that the strength evolution of the public has not been changed significantly by the anti. The anti's incompatibility to

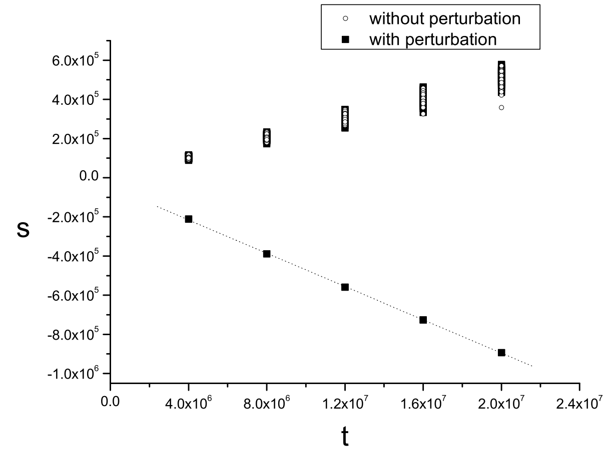

FIG. 5: strength evolution for N=50, M=50 and p=1.00 after $2.0{\times}10^7$ time steps. The dot line (with slop -0.0425) fits the strength evolution of the public anti, and blacksquare(■) represents the strength of the public, circle(○) denotes the strength evolution without the anti, under the same conditions. The data are drawn out at intervals of $4.0{\times}10^6$ time steps.

the public is aggravated. Recall that $<\gamma>$=0.67 in section 2, and suppose $<\gamma>_j\approx<\gamma>$ (for $j \neq i$). We could estimate the growth rate of the public strength as $2<\gamma>_j/N \approx 2<\gamma>/N$=$2\times 0.76/50$=0.0304. The slope of the anti's popularity evolution is -0.0425. It is reasonable to assume that $|\omega_{kj}| \ll |\omega_{ij}|$ for all j and $i \neq k$, then $\gamma_{ij} \approx -1$, and further $\partial s_i/\partial t$=$2/N$=-0.04, in fine agreement with the experimental results.

It must be stressed that the strength growth is actually nonlinear. As Fig. 6 indicates, the trace of the strength evolution behaves linear initially, but nonlinear afterwards. Experiments demonstrate that the strength evolution exhibits obvious nonlinear behaviors after sufficient time, whatever the initial state. When p=1.00, it has a widest linear zone, but finally, it splits. We would like to discuss the adaptive process of a student joining a class midway. Plotted in Fig. 8 (log-log scale) is the strength evolution of the system on the background of p=1.00. The old students' popularity grows linearly, even after inserting the new one. One could see the nonlinear growth for the new-coming pupil, whose popularity grows approaching the asymptote of the old students' strength evolution. The adaptive process for the new is quite short, thanks to the harmonious group.

IV. OUTLOOK AND APPLICATIONS

This paper is focused on discussing various perturbations and corresponding evolutions of our original model, which is proposed to study the student relationships in a class. Both experimental and analytical methods have been used to learn human relations in a local group. The generalization of the model to describe the real social networks is still a challenging problem. (to be continued.)

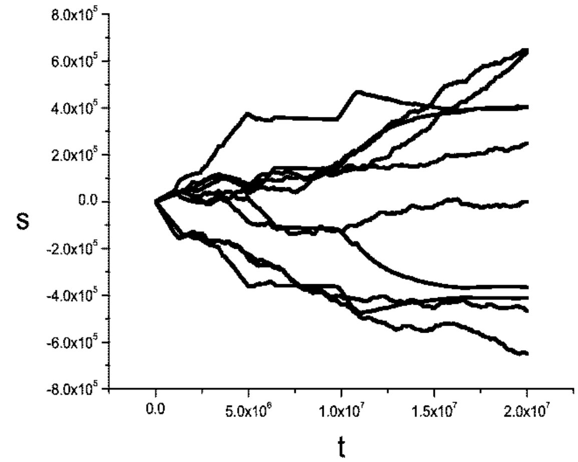

FIG. 6: strength evolution for N=10, p=0.00 after $2.0{\times}10^7$ time steps.

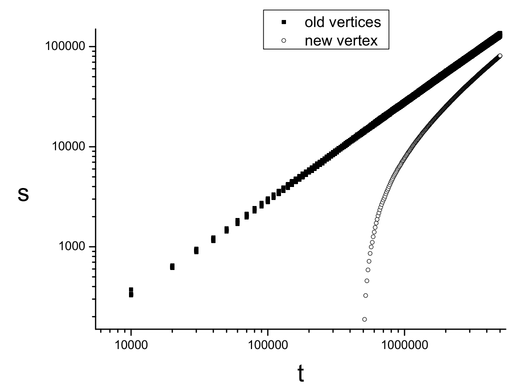

FIG. 7: strength evolution of a student joining midway: N=49 and p=1.00. The new student joins the class after $4.0{\times}10^5$ time steps, and his evolution is denoted by ♢. the strength evolution of old students are denoted by ■. We have extracted 5 from them as representatives.

Acknowledgments

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